90 Minutes—No Calculator

Note: In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

1. If $f(x) = e^{1/x}$, then $f'(x) =$

(A) $-\dfrac{e^{1/x}}{x^2}$ (B) $-e^{1/x}$ (C) $\dfrac{e^{1/x}}{x}$ (D) $\dfrac{e^{1/x}}{x^2}$ (E) $\dfrac{1}{x}e^{(1/x)-1}$

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2. $\int_0^3 (x+1)^{1/2}\,dx =$

(A) $\dfrac{21}{2}$ (B) 7 (C) $\dfrac{16}{3}$ (D) $\dfrac{14}{3}$ (E) $-\dfrac{1}{4}$

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3. If $f(x) = x + \dfrac{1}{x}$, then the set of values for which f increases is

(A) $(-\infty, -1] \cup [1, \infty)$ (B) $[-1, 1]$ (C) $(-\infty, \infty)$

(D) $(0, \infty)$ (E) $(-\infty, 0) \cup (0, \infty)$

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4. For what non-negative value of b is the line given by $y = -\dfrac{1}{3}x + b$ normal to the curve $y = x^3$?

(A) 0 (B) 1 (C) $\dfrac{4}{3}$ (D) $\dfrac{10}{3}$ (E) $\dfrac{10\sqrt{3}}{3}$

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5. $\int_{-1}^{2} \dfrac{|x|}{x}\,dx$ is

(A) –3 (B) 1 (C) 2 (D) 3 (E) nonexistent

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6. If $f(x) = \dfrac{x-1}{x+1}$ for all $x \neq -1$, then $f'(1) =$

(A) –1 (B) $-\dfrac{1}{2}$ (C) 0 (D) $\dfrac{1}{2}$ (E) 1

7. If $y = \ln\left(x^2 + y^2\right)$, then the value of $\frac{dy}{dx}$ at the point $(1,0)$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) undefined

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8. If $y = \sin x$ and $y^{(n)}$ means "the *n*th derivative of *y* with respect to *x*," then the smallest positive integer *n* for which $y^{(n)} = y$ is

(A) 2 (B) 4 (C) 5 (D) 6 (E) 8

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9. If $y = \cos^2 3x$, then $\frac{dy}{dx} =$

(A) $-6\sin 3x \cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$
(D) $6\cos 3x$ (E) $2\sin 3x \cos 3x$

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10. The length of the curve $y = \ln \sec x$ from $x = 0$ to $x = b$, where $0 < b < \frac{\pi}{2}$, may be expressed by which of the following integrals?

(A) $\int_0^b \sec x \, dx$

(B) $\int_0^b \sec^2 x \, dx$

(C) $\int_0^b (\sec x \tan x) \, dx$

(D) $\int_0^b \sqrt{1 + (\ln \sec x)^2} \, dx$

(E) $\int_0^b \sqrt{1 + \left(\sec^2 x \tan^2 x\right)} \, dx$

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11. Let $y = x\sqrt{1 + x^2}$. When $x = 0$ and $dx = 2$, the value of dy is

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

12. If n is a known positive integer, for what value of k is $\int_1^k x^{n-1}\,dx = \frac{1}{n}$?

(A) 0 (B) $\left(\frac{2}{n}\right)^{1/n}$ (C) $\left(\frac{2n-1}{n}\right)^{1/n}$

(D) $2^{1/n}$ (E) 2^n

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13. The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(4) - s(2)$?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

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14. If $x = t^2 - 1$ and $y = 2e^t$, then $\frac{dy}{dx} =$

(A) $\frac{e^t}{t}$ (B) $\frac{2e^t}{t}$ (C) $\frac{e^{|t|}}{t^2}$ (D) $\frac{4e^t}{2t-1}$ (E) e^t

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15. The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{x/2}$ is

(A) $\frac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$

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16. A series expansion of $\frac{\sin t}{t}$ is

(A) $1 - \frac{t^2}{3!} + \frac{t^4}{5!} - \frac{t^6}{7!} + \cdots$

(B) $\frac{1}{t} - \frac{t}{2!} + \frac{t^3}{4!} - \frac{t^5}{6!} + \cdots$

(C) $1 + \frac{t^2}{3!} + \frac{t^4}{5!} + \frac{t^6}{7!} + \cdots$

(D) $\frac{1}{t} + \frac{t}{2!} + \frac{t^3}{4!} + \frac{t^5}{6!} + \cdots$

(E) $t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots$

17. The number of bacteria in a culture is growing at a rate of $3,000e^{2t/5}$ per unit of time t. At $t=0$, the number of bacteria present was 7,500. Find the number present at $t=5$.

(A) $1,200e^2$ (B) $3,000e^2$ (C) $7,500e^2$ (D) $7,500e^5$ (E) $\frac{15,000}{7}e^7$

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18. Let g be a continuous function on the closed interval $[0,1]$. Let $g(0)=1$ and $g(1)=0$. Which of the following is NOT necessarily true?

(A) There exists a number h in $[0,1]$ such that $g(h) \geq g(x)$ for all x in $[0,1]$.

(B) For all a and b in $[0,1]$, if $a=b$, then $g(a)=g(b)$.

(C) There exists a number h in $[0,1]$ such that $g(h)=\frac{1}{2}$.

(D) There exists a number h in $[0,1]$ such that $g(h)=\frac{3}{2}$.

(E) For all h in the open interval $(0,1)$, $\lim_{x \to h} g(x) = g(h)$.

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19. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ II. $\sum_{n=1}^{\infty} \frac{1}{n}$ III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(A) I only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

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20. $\int x\sqrt{4-x^2}\,dx =$

(A) $\frac{\left(4-x^2\right)^{3/2}}{3}+C$ (B) $-\left(4-x^2\right)^{3/2}+C$ (C) $\frac{x^2\left(4-x^2\right)^{3/2}}{3}+C$

(D) $-\frac{x^2\left(4-x^2\right)^{3/2}}{3}+C$ (E) $-\frac{\left(4-x^2\right)^{3/2}}{3}+C$

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21. $\int_0^1 (x+1)e^{x^2+2x}\,dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

22. A particle moves on the curve $y = \ln x$ so that the x-component has velocity $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(1, 0)$. At time $t = 1$, the particle is at the point

(A) $(2, \ln 2)$ (B) $(e^2, 2)$ (C) $\left(\frac{5}{2}, \ln\frac{5}{2}\right)$

(D) $(3, \ln 3)$ (E) $\left(\frac{3}{2}, \ln\frac{3}{2}\right)$

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23. $\lim\limits_{h \to 0} \frac{1}{h} \ln\left(\frac{2+h}{2}\right)$ is

(A) e^2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) nonexistent

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24. Let $f(x) = 3x + 1$ for all real x and let $\varepsilon > 0$. For which of the following choices of δ is $|f(x) - 7| < \varepsilon$ whenever $|x - 2| < \delta$?

(A) $\frac{\varepsilon}{4}$ (B) $\frac{\varepsilon}{2}$ (C) $\frac{\varepsilon}{\varepsilon + 1}$ (D) $\frac{\varepsilon + 1}{\varepsilon}$ (E) 3ε

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25. $\int_0^{\pi/4} \tan^2 x \, dx =$

(A) $\frac{\pi}{4} - 1$ (B) $1 - \frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2} - 1$ (E) $\frac{\pi}{4} + 1$

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26. Which of the following is true about the graph of $y = \ln|x^2 - 1|$ in the interval $(-1, 1)$?

(A) It is increasing.
(B) It attains a relative minimum at $(0, 0)$.
(C) It has a range of all real numbers.
(D) It is concave down.
(E) It has an asymptote of $x = 0$.

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27. If $f(x) = \frac{1}{3}x^3 - 4x^2 + 12x - 5$ and the domain is the set of all x such that $0 \leq x \leq 9$, then the absolute maximum value of the function f occurs when x is

(A) 0 (B) 2 (C) 4 (D) 6 (E) 9

28. If the substitution $\sqrt{x} = \sin y$ is made in the integrand of $\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$, the resulting integral is

(A) $\int_0^{1/2} \sin^2 y\,dy$ (B) $2\int_0^{1/2} \frac{\sin^2 y}{\cos y}\,dy$ (C) $2\int_0^{\pi/4} \sin^2 y\,dy$

(D) $\int_0^{\pi/4} \sin^2 y\,dy$ (E) $2\int_0^{\pi/6} \sin^2 y\,dy$

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29. If $y'' = 2y'$ and if $y = y' = e$ when $x = 0$, then when $x = 1$, $y =$

(A) $\frac{e}{2}\left(e^2+1\right)$ (B) e (C) $\frac{e^3}{2}$ (D) $\frac{e}{2}$ (E) $\frac{\left(e^3-e\right)}{2}$

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30. $\int_1^2 \frac{x-4}{x^2}\,dx$

(A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

---

31. If $f(x) = \ln(\ln x)$, then $f'(x) =$

(A) $\frac{1}{x}$ (B) $\frac{1}{\ln x}$ (C) $\frac{\ln x}{x}$ (D) x (E) $\frac{1}{x\ln x}$

---

32. If $y = x^{\ln x}$, then y' is

(A) $\frac{x^{\ln x}\ln x}{x^2}$

(B) $x^{1/x}\ln x$

(C) $\frac{2x^{\ln x}\ln x}{x}$

(D) $\frac{x^{\ln x}\ln x}{x}$

(E) None of the above

33. Suppose that f is an odd function; i.e., $f(-x) = -f(x)$ for all x. Suppose that $f'(x_0)$ exists. Which of the following must necessarily be equal to $f'(-x_0)$?

(A) $f'(x_0)$

(B) $-f'(x_0)$

(C) $\dfrac{1}{f'(x_0)}$

(D) $-\dfrac{1}{f'(x_0)}$

(E) None of the above

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34. The average (mean) value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

(A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

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35. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \dfrac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\dfrac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\dfrac{8\pi}{3}$

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36. $\displaystyle\int_0^1 \frac{x+1}{x^2+2x-3}\,dx$ is

(A) $-\ln\sqrt{3}$ (B) $-\dfrac{\ln\sqrt{3}}{2}$ (C) $\dfrac{1-\ln\sqrt{3}}{2}$ (D) $\ln\sqrt{3}$ (E) divergent

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37. $\displaystyle\lim_{x\to 0} \frac{1-\cos^2(2x)}{x^2} =$

(A) –2 (B) 0 (C) 1 (D) 2 (E) 4

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38. If $\displaystyle\int_1^2 f(x-c)\,dx = 5$ where c is a constant, then $\displaystyle\int_{1-c}^{2-c} f(x)\,dx =$

(A) $5+c$ (B) 5 (C) $5-c$ (D) $c-5$ (E) –5

39. Let f and g be differentiable functions such that

$$f(1)=2, \quad f'(1)=3, \quad f'(2)=-4,$$

$$g(1)=2, \quad g'(1)=-3, \quad g'(2)=5.$$

If $h(x)=f\left(g(x)\right)$, then $h'(1)=$

(A) –9 (B) –4 (C) 0 (D) 12 (E) 15

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40. The area of the region enclosed by the polar curve $r=1-\cos\theta$ is

(A) $\frac{3}{4}\pi$ (B) π (C) $\frac{3}{2}\pi$ (D) 2π (E) 3π

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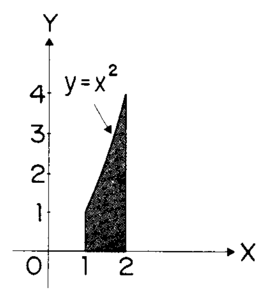
41. Given $f(x)=\begin{cases} x+1 & \text{for } x<0, \\ \cos\pi x & \text{for } x\geq 0, \end{cases}$ $\int_{-1}^{1} f(x)\,dx=$

(A) $\frac{1}{2}+\frac{1}{\pi}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}-\frac{1}{\pi}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2}+\pi$



42. Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x=\frac{4}{3}$ and $x=\frac{5}{3}$.

(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

43. $\int \arcsin x \, dx =$

(A) $\sin x - \int \frac{x \, dx}{\sqrt{1-x^2}}$

(B) $\frac{(\arcsin x)^2}{2} + C$

(C) $\arcsin x + \int \frac{dx}{\sqrt{1-x^2}}$

(D) $x \arccos x - \int \frac{x \, dx}{\sqrt{1-x^2}}$

(E) $x \arcsin x - \int \frac{x \, dx}{\sqrt{1-x^2}}$

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44. If f is the solution of $x f'(x) - f(x) = x$ such that $f(-1) = 1$, then $f\left(e^{-1}\right) =$

(A) $-2e^{-1}$ (B) 0 C) e^{-1} (D) $-e^{-1}$ (E) $2e^{-2}$

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45. Suppose $g'(x) < 0$ for all $x \geq 0$ and $F(x) = \int_0^x t \, g'(t) \, dt$ for all $x \geq 0$. Which of the following statements is FALSE?

(A) F takes on negative values.

(B) F is continuous for all $x > 0$.

(C) $F(x) = x \, g(x) - \int_0^x g(t) \, dt$

(D) $F'(x)$ exists for all $x > 0$.

(E) F is an increasing function.

1973 AB

1. E
2. E
3. B
4. A
5. A
6. D
7. B
8. B
9. A
10. C
11. B
12. C
13. D
14. D
15. C
16. C
17. C
18. D
19. D
20. D
21. B
22. B
23. C
24. B
25. B
26. E
27. E
28. C
29. C
30. B
31. D
32. D
33. A
34. C
35. C
36. A
37. A
38. B
39. B
40. E
41. D
42. D
43. E
44. B
45. C

1973 BC

1. A
2. D
3. A
4. C
5. B
6. D
7. D
8. B
9. A
10. A
11. E
12. D
13. D
14. A
15. C
16. A
17. C
18. D
19. D
20. E
21. B
22. C
23. C
24. A
25. B
26. D
27. E
28. C
29. A
30. B
31. E
32. C
33. A
34. C
35. C
36. E
37. E
38. B
39. D
40. C
41. D
42. D
43. E
44. A
45. E

1. A $f'(x)=e^{\frac{1}{x}}\cdot\dfrac{d\left(\frac{1}{x}\right)}{dx}=e^{\frac{1}{x}}\left(-\dfrac{1}{x^2}\right)=-\dfrac{e^{\frac{1}{x}}}{x^2}$

2. D $\int_0^3 (x+1)^{\frac{1}{2}}\,dx=\dfrac{2}{3}(x+1)^{\frac{3}{2}}\Big|_0^3=\dfrac{2}{3}\left(4^{\frac{3}{2}}-1^{\frac{3}{2}}\right)=\dfrac{2}{3}(8-1)=\dfrac{14}{3}$

3. A $f'(x)=1-\dfrac{1}{x^2}=\dfrac{(x+1)(x-1)}{x^2}$. $f'(x)>0$ for $x<-1$ and for $x>1$.

   f is increasing for $x\le -1$ and for $x\ge 1$.

4. C The slopes will be negative reciprocals at the point of intersection.

   $3x^2=3\Rightarrow x=\pm1$ and $x\ge 0,$ thus $x=1$ and the y values must be the same at $x=1$.

   $-\dfrac{1}{3}+b=1\Rightarrow b=\dfrac{4}{3}$

5. B $\int_{-1}^{2}\dfrac{|x|}{x}\,dx=\int_{-1}^{0}-1\,dx+\int_0^2 dx=-1+2=1$

6. D $f'(x)=\dfrac{(1)(x+1)-(x-1)(1)}{(x+1)^2}$, $f'(1)=\dfrac{2}{4}=\dfrac{1}{2}$

7. D $\dfrac{dy}{dx}=\dfrac{2x+2y\cdot\frac{dy}{dx}}{x^2+y^2}$ at $(1,0)\Rightarrow y'=\dfrac{2}{1}=2$

8. B $y=\sin x$, $y'=\cos x$, $y''=-\sin x$, $y'''=-\cos x$, $y^{(4)}=\sin x$

9. A $y'=2\cos 3x\cdot\dfrac{d}{dx}(\cos 3x)=2\cos 3x\cdot(-\sin 3x)\cdot\dfrac{d}{dx}(3x)=2\cos 3x\cdot(-\sin 3x)\cdot 3$

   $y'=-6\sin 3x\cos 3x$

10. A $L=\int_0^b \sqrt{1+(y')^2}\,dx=\int_0^b \sqrt{1+\left(\frac{\sec x\tan x}{\sec x}\right)^2}\,dx$

$=\int_0^b \sqrt{1+(\tan x)^2}\,dx=\int_0^b \sqrt{\sec^2 x}\,dx=\int_0^b \sec x\,dx$

11. E $dy=\left(x\cdot\frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}}(2x)+\left(1+x^2\right)^{\frac{1}{2}}\right)dx;\ dy=(0+1)(2)=2$

12. D $\frac{1}{n}=\int_1^k x^{n-1}\,dx=\left.\frac{x^n}{n}\right|_1^k \Rightarrow \frac{1}{n}=\frac{k^n}{n}-\frac{1}{n};\ \ \frac{k^n}{n}=\frac{2}{n}\Rightarrow k=2^{\frac{1}{n}}$

13. D $v(t)=8t-3t^2+C$ and $v(1)=25\Rightarrow C=20$ so $v(t)=8t-3t^2+20$.

$s(4)-s(2)=\int_2^4 v(t)\,dt=\left.\left(4t^2-t^3+20t\right)\right|_2^4=32$

14. A $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2e^t}{2t}=\frac{e^t}{t}$

15. C $\text{Area}=\int_0^2 e^{\frac{1}{2}x}\,dx=\left.2e^{\frac{1}{2}x}\right|_0^2=2(e-1)$

16. A $\sin t=t-\frac{t^3}{3!}+\frac{t^5}{5!}-\frac{t^7}{7!}+\cdots\Rightarrow\frac{\sin t}{t}=1-\frac{t^2}{3!}+\frac{t^4}{5!}-\frac{t^6}{7!}+\cdots$

17. C $\frac{dN}{dt}=3000e^{\frac{2}{5}t}$, $N=7500e^{\frac{2}{5}t}+C$ and $N(0)=7500\Rightarrow C=0$

$N=7500e^{\frac{2}{5}t}$, $N(5)=7500e^2$

18. D D could be false, consider $g(x)=1-x$ on $[0,1]$. A is true by the Extreme Value Theorem, B is true because g is a function, C is true by the Intermediate Value Theorem, and E is true because g is continuous.

19. D I is a convergent p-series, $p = 2 > 1$
II is the Harmonic series and is known to be divergent,
III is convergent by the Alternating Series Test.

20. E $\int x\sqrt{4-x^2}\,dx = -\frac{1}{2}\int \left(4-x^2\right)^{\frac{1}{2}}(-2x\,dx) = -\frac{1}{2}\cdot\frac{2}{3}\left(4-x^2\right)^{\frac{3}{2}} + C = -\frac{1}{3}\left(4-x^2\right)^{\frac{3}{2}} + C$

21. B $\int_0^1 (x+1)e^{x^2+2x}\,dx = \frac{1}{2}\int_0^1 e^{x^2+2x}\left((2x+2)\,dx\right) = \frac{1}{2}\left(e^{x^2+2x}\right)\Big|_0^1 = \frac{1}{2}\left(e^3 - e^0\right) = \frac{e^3-1}{2}$

22. C $x'(t) = t+1 \Rightarrow x(t) = \frac{1}{2}(t+1)^2 + C$ and $x(0) = 1 \Rightarrow C = \frac{1}{2} \Rightarrow x(t) = \frac{1}{2}(t+1)^2 + \frac{1}{2}$

$x(1) = \frac{5}{2},\ y(1) = \ln\frac{5}{2};\qquad \left(\frac{5}{2}, \ln\frac{5}{2}\right)$

23. C $\lim_{h\to 0}\frac{\ln(2+h)-\ln 2}{h} = f'(2)$ where $f(x) = \ln x$; $\quad f'(x) = \frac{1}{x} \Rightarrow f'(2) = \frac{1}{2}$

24. A This item uses the formal definition of a limit and is no longer part of the AP Course Description. $\left|f(x)-7\right| = \left|(3x+1)-7\right| = \left|3x-6\right| = 3\left|x-2\right| < \varepsilon$ whenever $\left|x-2\right| < \frac{\varepsilon}{3}$.

Any $\delta < \frac{\varepsilon}{3}$ will be sufficient and $\frac{\varepsilon}{4} < \frac{\varepsilon}{3}$, thus the answer is $\frac{\varepsilon}{4}$.

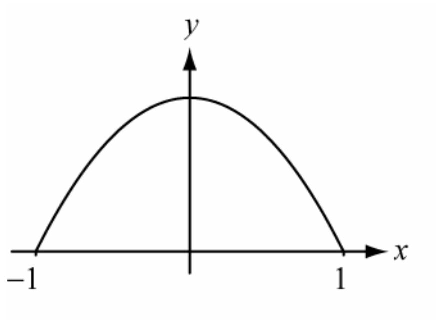
25. B $\int_0^{\frac{\pi}{4}} \tan^2 x\,dx = \int_0^{\frac{\pi}{4}}\left(\sec^2 x - 1\right)dx = \left(\tan x - x\right)\Big|_0^{\pi/4} = 1 - \frac{\pi}{4}$

26. D For x in the interval (–1, 1), $g(x) = \left|x^2-1\right| = -(x^2-1)$ and so $y = \ln g(x) = \ln(-(x^2-1))$.

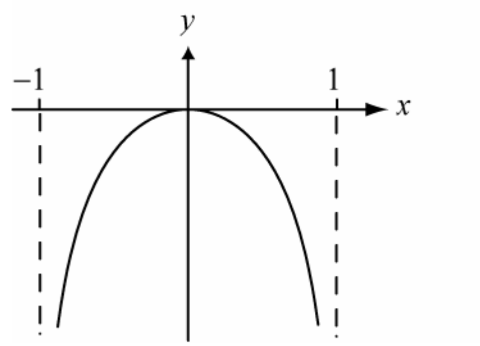
Therefore

$$y' = \frac{2x}{x^2-1},\quad y'' = \frac{\left(x^2-1\right)(2) - (2x)(2x)}{\left(x^2-1\right)^2} = \frac{-2x^2-2}{\left(x^2-1\right)^2} < 0$$

Alternative graphical solution: Consider the graphs of $g(x)=\left|x^2-1\right|$ and $\ln g(x)$.



$g(x)=\left|x^2-1\right|$



$\ln\left|x^2-1\right|$

concave down

27. E $\quad f'(x)=x^2-8x+12=(x-2)(x-6)$; the candidates are: $x=0,2,6,9$

x	0	2	6	9
$f(x)$	−5	$17/3$	−5	22

the maximum is at $x=9$

28. C $\quad x=\sin^2 y \Rightarrow dx=2\sin y\cos y\,dy$; when $x=0,\ y=0$ and when $x=\frac{1}{2},\ y=\frac{\pi}{4}$

$$\int_0^{\frac{1}{2}}\frac{\sqrt{x}}{\sqrt{1-x}}\,dx=\int_0^{\frac{\pi}{4}}\frac{\sin y}{\sqrt{1-\sin^2 y}}\cdot 2\sin y\cos y\,dy=\int_0^{\frac{\pi}{4}}2\sin^2 y\,dy$$

29. A $\quad$ Let $z=y'$. Then $z=e$ when $x=0$. Thus $y''=2y'\Rightarrow z'=2z$. Solve this differential equation.

$z=Ce^{2x};\ e=Ce^0\Rightarrow C=e\Rightarrow y'=z=e^{2x+1}$. Solve this differential equation.

$$y=\frac{1}{2}e^{2x+1}+K;\ e=\frac{1}{2}e^1+K\Rightarrow K=\frac{1}{2}e;\ y=\frac{1}{2}e^{2x+1}+\frac{1}{2}e,\ y(1)=\frac{1}{2}e^3+\frac{1}{2}e=\frac{1}{2}e\left(e^2+1\right)$$

Alternative Solution: $y''=2y'\Rightarrow y'=Ce^{2x}=e\cdot e^{2x}$. Therefore $y'(1)=e^3$.

$y'(1)-y'(0)=\int_0^1 y''(x)dx=\int_0^1 2y'(x)dx=2y(1)-2y(0)$ and so

$$y(1)=\frac{y'(1)-y'(0)+2y(0)}{2}=\frac{e^3+e}{2}.$$

30. B $\int_1^2 \frac{x-4}{x^2}dx = \int_1^2 \left(\frac{1}{x} - 4x^{-2}\right)dx = \left(\ln x + \frac{4}{x}\right)\Big|_1^2 = (\ln 2 + 2) - (\ln 1 + 4) = \ln 2 - 2$

31. E $f'(x) = \dfrac{\frac{d}{dx}(\ln x)}{\ln x} = \dfrac{\frac{1}{x}}{\ln x} = \dfrac{1}{x \ln x}$

32. C Take the log of each side of the equation and differentiate. $\ln y = \ln x^{\ln x} = \ln x \cdot \ln x = (\ln x)^2$

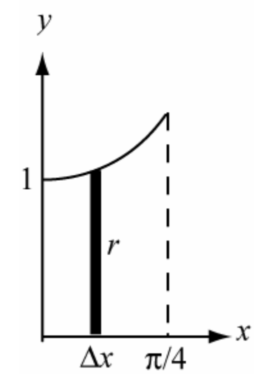
$$\frac{y'}{y} = 2\ln x \cdot \frac{d}{dx}(\ln x) = \frac{2}{x}\ln x \Rightarrow y' = x^{\ln x}\left(\frac{2}{x}\ln x\right)$$

33. A $f(-x) = -f(x) \Rightarrow f'(-x)\cdot(-1) = -f'(x) \Rightarrow f'(-x) = -f'(x)$ thus $f'(-x_0) = -f'(x_0)$.

34. C $\frac{1}{2}\int_0^2 \sqrt{x}\,dx = \frac{1}{2}\cdot\frac{2}{3}x^{\frac{3}{2}}\Big|_0^2 = \frac{1}{3}\cdot 2^{\frac{3}{2}} = \frac{2}{3}\sqrt{2}$

35. C Washers: $\sum \pi r^2 \Delta x$ where $r = y = \sec x$.

$$\text{Volume} = \pi\int_0^{\frac{\pi}{4}} \sec^2 x\,dx = \pi \tan x\Big|_0^{\frac{\pi}{4}} = \pi\left(\tan\frac{\pi}{4} - \tan 0\right) = \pi$$



36. E $\int_0^1 \frac{x+1}{x^2+2x-3}dx = \frac{1}{2}\lim_{L\to 1^-}\int_0^L \frac{2x+2}{x^2+2x-3}dx = \frac{1}{2}\lim_{L\to 1^-} \ln\left|x^2+2x-3\right|\Big|_0^L$

$$= \frac{1}{2}\lim_{L\to 1^-}\left(\ln\left|L^2+2L-3\right| - \ln\left|-3\right|\right) = -\infty. \quad \text{Divergent}$$

37. E $\lim_{x\to 0}\frac{1-\cos^2 2x}{x^2} = \lim_{x\to 0}\frac{\sin^2 2x}{x^2} = \lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\frac{\sin 2x}{2x}\cdot 4 = 1\cdot 1\cdot 4 = 4$

38. B Let $z = x - c$. $5 = \int_1^2 f(x-c)\,dx = \int_{1-c}^{2-c} f(z)\,dz$

39. D $h'(x) = f'(g(x))\cdot g'(x)$; $h'(1) = f'(g(1))\cdot g'(1) = f'(2)\cdot g'(1) = (-4)(-3) = 12$

40. C Area $=\frac{1}{2}\int_0^{2\pi}(1-\cos\theta)^2 d\theta=\int_0^{\pi}\left(1-2\cos\theta+\cos^2\theta\right)d\theta$; $\cos^2\theta=\frac{1}{2}(1+\cos 2\theta)$

$$\text{Area}=\int_0^{\pi}\left(1-2\cos\theta+\frac{1}{2}(1+\cos 2\theta)\right)d\theta=\left(\frac{3}{2}\theta-2\sin\theta+\frac{1}{4}\sin 2\theta\right)\Bigg|_0^{\pi}=\frac{3}{2}\pi$$

41. D $\int_{-1}^{1} f(x)\,dx=\int_{-1}^{0}(x+1)\,dx+\int_0^1\cos(\pi x)\,dx$

$$=\frac{1}{2}(x+1)^2\Bigg|_{-1}^{0}+\frac{1}{\pi}\sin(\pi x)\Bigg|_0^1=\frac{1}{2}+\frac{1}{\pi}\left(\sin\pi-\sin 0\right)=\frac{1}{2}$$

42. D $\Delta x=\frac{1}{3};\ T=\frac{1}{2}\cdot\frac{1}{3}\left(1^2+2\left(\frac{4}{3}\right)^2+2\left(\frac{5}{3}\right)^2+2^2\right)=\frac{127}{54}$

43. E Use the technique of antiderivatives by part:

$u=\sin^{-1}x \qquad dv=dx$

$du=\frac{dx}{\sqrt{1-x^2}} \qquad v=x$

$$\int \sin^{-1}x\,dx=x\sin^{-1}x-\int\frac{x}{\sqrt{1-x^2}}\,dx$$

44. A Multiply both sides of $x=xf'(x)-f(x)$ by $\frac{1}{x^2}$. Then $\frac{1}{x}=\frac{xf'(x)-f(x)}{x^2}=\frac{d}{dx}\left(\frac{f(x)}{x}\right)$.

Thus we have $\frac{f(x)}{x}=\ln|x|+C\Rightarrow f(x)=x\left(\ln|x|+C\right)=x\left(\ln|x|-1\right)$ since $f(-1)=1$.

Therefore $f(e^{-1})=e^{-1}\left(\ln\left|e^{-1}\right|-1\right)=e^{-1}(-1-1)=-2e^{-1}$

This was most likely the solution students were expected to produce while solving this problem on the 1973 multiple-choice exam. However, the problem itself is not well-defined. A solution to an initial value problem should be a function that is differentiable on an interval containing the initial point. In this problem that would be the domain $x<0$ since the solution requires the choice of the branch of the logarithm function with $x<0$. Thus one cannot ask about the value of the function at $x=e^{-1}$.

45. E $F'(x)=xg'(x)$ with $x\ge 0$ and $g'(x)<0\Rightarrow F'(x)<0\Rightarrow F$ is not increasing.